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Teaching undergraduate mathematics on the internet. Part 2: Attributes and possibilities

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Abstract
Internet education in mathematics is developing as a new mode of teaching with its own characteristics and possibilities, different from the traditional way of teaching. In a study presented in two parts, we attempt to capture the world of internet teaching of undergraduate mathematics. In the first part of the study (Engelbrecht and Harding, 2004), we attempt to create some order in the huge number of activities that are available on the web with a graphical classification of the different types of web courses and discuss some of the technologies involved. In this, the second part of the study, we discuss attributes and implications of this mode of teaching/learning mathematics, also mentioning some of the benefits and concerns. Speculation about the future in such a fluid environment is both risky and fascinating but we nevertheless envisage some possible future trends. Research on this mode of teaching is sparse and open research questions are plentiful. We list possible research issues.

1. Pedagogical Issues

Although much has been done to develop a pedagogy for distance learning and also for computer-based learning, a pedagogy for driving online courses in mathematics is still only in its development phase. Hopper (2001) believes that "an online pedagogy is not yet in sight". Most of the research on distance learning was done before the internet was introduced into higher education and had methodologies as subject that are essentially different from those that are employed in web-based courses. Hopper (2001) mentions that

> Internet teaching is so different from any of the categories of distance learning that preceded it that it is essentially a practice without research foundation.

Online learning certainly has many facets that are not present in traditional teaching. However, the basic cognition principles of learning mathematics should also apply in this environment. Elements of the two most dominant cognition theories that have been applied to mathematics teaching (Crowe and Zand, 2001a) are *constructivism* (Bruner, 1990) and *socio-culturalism* (Vygotsky, 1978, 1987). These should also be applicable to this mode of teaching.

Although most of this theory has been developed in general and not for mathematics in particular, Selden and Selden (1997) agree that as far as mathematics is concerned knowledge is acquired by construction, not by transmission alone and that the process of knowledge acquisition is constrained both internally, by what one already knows, and externally, by cultural artefacts such as shared language and notation.

On the other hand, Czerniewicz (2001) warns that in a web-based course, one should not take constructivism too far, learners cannot be expected to generate their own ways of collaborating, for instance, as it will take more time than what they can afford. They benefit from having some clear guidance about how to participate in the learning situation. Practicing constructivism also does not necessarily imply that the student has to discover the merits of all the different components of the course such as online chat rooms, etc. She claims that it is extremely difficult to design an online model that is purely learner centred and is solely based on discovery and she advises that a guided construction model of learning providing structured ways of collaboration and solving problems is probably more appropriate.

In developing an online course in mathematics, one should take notice of these pedagogical issues and of the experience of people like Czerniewicz (2001) who suggests

> ...a subtle and complex process of course design where tutors and course designers create and develop an environment, a framework that enables and facilitates a range of learner centred activities which build on their existing knowledge and which encourages and shapes interactivity of purposeful kind.

It is clear that although little has been done in developing a pedagogy for online mathematics courses, there are some clear guidelines. Care should be taken to have a sound balance between teacher and learner centred activities and that interaction should be carefully planned; interaction between learner and content, between learner and instructor and between learner and learner.

Badrul H. Khan, in his well-known book on web-based instruction (Khan, 1997), emphasises that a meaningful learning environment should be created that fosters and supports learning. In this he is supported by Czerniewicz (2001), as mentioned above.

When starting out on online education, in the absence of the knowledge of what will work and what not and with no real online pedagogy available, many teachers will try to merely convert their traditional courses to the internet.

Using Schoenfeld's (1988) analogy, this comes as no surprise: Recall that the first "horseless carriages" looked just like carriages pulled by horses, simply because carriages were people's models of "moving vehicles". It took a while for cars to evolve their own shapes.

First generation web courses are being criticised for the absence of a sound underlying pedagogy and for employing dubious instructional strategies (Firdyiwek, 1999). Even excellent lecture driven courses become tedious and ineffectual when converted to electronic page turners. In many of the first generation web-based courses only the lower cognitive levels are addressed (Myers, 1999).

When teachers attempt to *convert* their lecture driven courses to the internet they are quick to note that much is lost in doing a literal conversion into a non-verbal course without the possibility of monitoring student comprehension in the conventional way. The difference between being physically present at a *good* lecture and reading the transcribed text of the same lecture is vast, and this is the gap that online course developers perceive (Hopper, 2001). Kawski (2003) has the same concern:

> … does the *story* of math still come across when classes move on-line? I like Poincaré who said: "Science is built up of facts as a house is built of stones; but an accumulation of facts is no more a science as a heap of stones is a house". One might think that the ultra-linked www would/could provide a great template for students making the associations between the facts (recipes) but my fear is that the presentations that I have seen on-line invite many a user to just pick the quick useful formula and completely miss out on the mortar.

Hopper (2001) questions whether the internet provides an environment comparable to the traditional classroom in that it consists almost entirely of typed text, depriving students of eye contact, body language and other human factors. He claims

> Even in a classroom wherein only the teacher speaks, there is a recognition of being physically present, of mutual awareness, and the student who merely listens attentively may in fact experience a highly intimate and satisfying learning and social transaction.

It is questionable whether this can be authentically replicated by the internet. However, research by Hopper and Harmon (2000) indicates that

> …the basic attributes of effective classroom teaching, such as rapid feedback, time on task and content expertise are the very factors that define current generation online courses of exemplary quality.

There exist web courses that are still based on lectures that are videotaped and students watch the lectures when and where they can, re-viewing the same part as many times as they want. This teaching style is not much different from the instructivist style of teaching - in fact, one could argue that web-based courses such as these could provoke a return to a backward pedagogy, with learners' participation reduced to reading and individual work on exercises.

2. Attributes of online learning settings

Sims et al (2001) provide a procedure by which online courses can be proactively evaluated.

> The essence of this concept is that by first considering the complex interactions between educational design and online environments, designers and developers with new or limited skills in online learning will reduce the risk of producing poor-quality or ineffective materials as well as the likelihood of critical, negative evaluation.

They emphasise that teachers should have total clarity about their strategic intent, should consider whether and how they are going to publish content and carefully consider the learning and the interface designs. Furthermore they should consider the interactivity of the medium, plan the way of student support and decide on appropriate assessment strategies.

Perhaps the way to begin is that teachers should begin tentatively using the web to supplement a traditional face-to-face course with a course website which initially may only contain the course information and perhaps some content. As the teacher becomes more at ease with the internet as part of the course the amount of web-based components in the course may gradually increase.

Some of the most serious errors in the educational design of web-based courses have been identified by Stiles (2000) and include

- failure to engage the learner
- confusing interactivity with engagement
- focussing on content rather than outcomes
- mirroring traditional didactic approaches on the technology
- failure to recognise the social nature of learning.

There is also some danger that there sometimes is an understandable desire to create contents using all available multi-media. This is first of all costly and secondly since most students have regular exposure to commercially developed computer games with far superior use of multi-media, they will hardly be impressed by even expensive and time consuming efforts by teachers in this regard. Stiles (2000) stresses that

> …the use of multi-media should focus on its value in the learning context, rather than to desire to excite with its "richness".

Allen et al (1998) go so far as to say that an online mathematics course must be the teacher, the mentor, the facilitator, the comforter and the threat. The course must do many things that mimic human interaction. To the student it must converse, engage, entertain, encourage, challenge and sympathise.

A number of studies have been done that describe the attributes of effective online learning settings, unfortunately most of them outside mathematics (e.g. Reeves & Reeves, 1997; Herrington et al, 2001; Swan et al, 2000; Porter, 1997; Sims et al, 2001; Tan and Hung, 2002; Berge, 1999). Allen (2001) and Engelbrecht and Harding (2001a, 2001b) have developed and presented web-based calculus courses for a number of years and also identified some attributes that such a course should strive for. We discuss some of these desirable components addressed by one or more of these studies.

Instructor facilitation

Engelbrecht and Harding (2001a, 2001b) describe how a web-based mathematics course can be dynamically run by the instructor playing the role of a "metronome" for the course. They distinguish between a dynamic course and a static course, the latter that could in essence also be run from a compact disc whenever the student wishes to do so. In such a dynamic course, instructors need to guide students' participation by interacting with them and facilitating online communities. They should also provide reassurance to students that they are doing the right things (Swan et al, 2000). Students should be brought into contact with teachers electronically to share ideas (Porter, 1997) and instructors should serve as task managers for administrative control, mentorship or guidance (Tan and Hung, 2002). More so than in a conventional contact course, the instructor's biggest task is in developing the course and preparing the website rather than being actively involved in presenting the course.

Communication opportunities

Communication opportunities are essential especially in a conceptual subject such as mathematics. Provision should be made for asynchronous and/or synchronous communication between participants through communication or conversation tools for co-construction of knowledge and sharing of ideas (Berge, 1999; Tan and Hung, 2002). This issue is discussed in detail in the next section.

Collaboration opportunities

Opportunities for collaboration should be provided (Herrington et al, 2001) using e.g. discussion groups (Berge, 1999). This issue is also discussed in detail in the next section.

Cognitive tools

Appropriate cognitive tools such as visualisation tools and information gathering tools should be provided (Tan and Hung, 2002). In a mathematical context, animations (animated GIF images or Java applets) have not yet become an *essential* course feature, although students do like them and they can very useful (Allen, 2001).

Internet resources

Resources such as the content and information that are provided for the learners should be accessible, current, should include purposeful use of media and be inclusive (Herrington et al, 2001). Links to other internet resources enriches a course (Allen, 2001) and offers exploration opportunities. In mathematics numerous dedicated (and well constructed) resource or content (textbook) sites (Engelbrecht and Harding, 2004) are available that can be used in conjunction with a web-based presentation with the advantage of saving on labour and cost.

Appropriate interface

Students should feel at ease with the course interface, they should know how to navigate through the interface and the interface should be consistent (Swan et al, 2000). The interface should be reliable and robust (Herrington et al, 2001). Allen (2001) also feels very strongly about the quality of the mathematical symbols on the website. While mathematics instructors have little difficulty in reading mathematics in almost any form, the beginner does have trouble. For these students, the presentation should preferably look as close to the textbook as possible. Allen (2001) emphasises the importance of graphical illustrations, stating that although graphics have emerged in mathematical textbooks in abundance only in the last half of the twentieth century

> … they [graphics] are today regarded as essential. In that connection students, having experience with the most cutting edge programmes today, are again the experts. The graphics must be excellent.

Online assessment

Assessment should be part of the design of the course (Porter, 1997). This could take the form of interactive quizzes and examinations including complete solutions to examples and exercises and question-answer notes (Allen, 2001). This issue is also addressed in detail in a later section.

3. Communication, interaction and collaboration

Distinction is made between synchronous (same time) and asynchronous (any time) communication tools in education. Examples of synchronous communication on the internet include multi-user domains such as voice and video conferencing, shared whiteboards and live presentation tools, application sharing, live assessment, chat rooms, web Safari (leading a live web browsing session) and breakout rooms for smaller groups.

Asynchronous communication does not require of teachers and students to be "present" simultaneously at a specific time or place for the activities. Asynchronous interaction involves parties communicating over elapsed time, usually in a text format, although not necessarily. Typical examples are e-mail and discussion forums. Asynchronous interaction could also include group project activity, assessments, surveys, votes, etc. These activities may be completely open-ended or may be constrained with a defined start or end time. Using asynchronous communication, students can work at their own convenience where and when they prefer. The key benefit of asynchronous interaction is its flexibility and ability to fit into everyone's working day. Asynchronous communication is sometimes called the "great equaliser" (Wepner and Mobley, 1998).

The importance of communication in learning mathematics is accepted worldwide and at many institutions teachers are evaluated also by their ability to foster communication opportunities with their students. This is no different when presenting mathematics on the internet. As an example we quote Hawisher and Pemberton (1997) who are of the opinion that the success that they had with their online courses could be because of the online discussions. They required their students to participate in online discussion twice a week. Their contributions are evaluated and these contribute to their semester marks.

Whether an online course in mathematics presents enough opportunity and facilities for communication is debatable. A course presented within a VLE provides the communication tools but students still need to be encouraged to communicate and ideally the course should be structured to necessitate communication. When designing an online offering, planning the communications to support learners can be just as important as designing the course material and presentation of that material. Unless students feel connected and supported, they will easily become frustrated when they encounter difficulties. Due to boredom, frustration or interruptions, completion rates for poorly developed asynchronous web-based training courses are often very low. According to Jones (2002) including the right kinds of communications in the course is the best way to ensure that students remain involved and "stay the course".

Czerniewicz (2001) agrees that an online course gives you speedier access to other people involved in networked learning but stresses that access alone does not mean that a meaningful conversation will take place, nor that useful interaction will happen. She argues that wherever possible, online courses should include a face-to-face component to maximise the benefits of networked learning and to minimise the problems. In her experience,

> … successful collaborative activities can rarely be left to learners and require careful planning, clear purpose and appropriate facilitation.

Not everyone agrees. Crowe and Zand (2000) argue that since mathematics is less subjective than other subjects, debate and interpretation play a lesser role. Furthermore, mathematics is claimed to be inherently less verbal than other subjects. They use these arguments to claim that a

> distance learner need not be at a significant disadvantage compared to a campus student provide that (s)he has the capability to communicate symbolically and pictorially with her/his fellow students.

The opinion that the internet per se does not encourage communication and collaboration and can only be fostered by aptly structured course activities is supported by Ng (2001). In a course in which e-mail was used to foster collaboration he states that most messages sent out by students reach the tutor but not their fellow students. Some students experience anxiety in the communication process, especially after having sent a message to the teacher. He further concludes that the sole use of e-mail as a tool to foster collaboration was not successful in his study.

> …most students considered that the major function of the online communication tool is to facilitate them to ask their tutor questions. Tutors are treated as experts in the field and their role is to transfer knowledge. This is clearly a very teacher-centred approach in which collaborative learning is unlikely to occur.

Group work and collaboration in the online environment has become a well-researched topic in recent years. Collaborative learning is embedded in constructivism. In the collaborative learning environment, small groups provide the social context and members create a community of learners whose goal is to construct knowledge thought common effort. In this new situation both teacher and student roles have to be redefined. (Miskulin et al, 2002). Although it is not yet clear from research that interaction improves the quality of learning in distance education, Kearsley (1995) states that research does indicate that interaction is important for learning satisfaction and that it assists in maintaining the persistence of distance students.

An important area in this field is what Johnson (2001) refers to as *communities of practice.*

> Communities of practice employ active participation and decision making by individuals, as opposed to separated decision-making that is present in traditional organizations. … The learning that evolves from these communities is collaborative, in which the collaborative knowledge of the community is greater than any individual knowledge.

The question arises whether current web-based environments can be used successfully to set up communities of practice that operate as learning entities. Research (Parloff and Pratt, 1999) shows that in some instances virtual communities have a key characteristic that is especially conducive for communities of practice to emerge and this includes the lack of traditional group norms like voice, visible reactions like approval or disapproval.

Miskulin et al (2002) describe an innovative example of collaborative learning in mathematics in a web-based environment. A multimedia interactive environment called E-TEAM is created that provides a favourable context for collaborative learning and shared knowledge, through electronic communication. Graphs can be retrieved, comments can be made on these objects in recorded voice on the screen as if the user were face-to-face with somebody else explaining a certain subject. The information is compacted and sent to the receiver via e-mail. The person who receives the file can open it and edit it and make comments about its objects in an interactive way.

Research on collaborative work within a virtual learning environment by Engelbrecht and Harding (2002a) shows that this is an effective strategy for communication between students within what could be experienced as an impersonal environment by students.

In a visionary effort, the use of a palmtop environment for mathematics communication over the internet is described by Isoda et al (2002). It was found that it is not easy for novice users of the environment to communicate and collaborate on mathematics but it is possible even in a small palmtop environment if you are accustomed to the environment and it is well-designed.

4. Assessment

When teaching by means of technology, such as in a web-based environment, the foundations are laid for assessing online and it seems almost natural to assess online. If technology is incorporated in the presentation of the course, it makes little sense to avoid technology in the assessment part of the course. In a study involving mathematics students Smith and Wood (2000) claim that

> …appropriate assessment methods are of major importance in encouraging students to adopt successful approaches to their learning. Changing teaching without due attention to assessment is not sufficient.

It is important to note that online assessment is not only used to test the lower level cognitive skills. The traditional perception is that multiple-choice questions can only be used for testing lower level cognitive skills. This is not true, according to Hibberd (1996)

> … they can be implemented to measure deeper understanding if questions are imaginatively constructed.

Engelbrecht and Harding did a study (2003a, 2003b) in which they compare online and paper-based assessment in a web-based environment in mathematics. They recommend that online assessment should be a component of the total assessment spectrum combined with traditional paper-based assessment and come to the conclusion that standards can be maintained using a combination of online and paper-based assessment in mathematics.

In a working group discussion on web-delivered assessment (Booth et al, 2001) at the 27th Undergraduate Mathematics Teaching Conference in Birmingham in September 2001, the group identified a number of advantages of web-based assessment, including

- Formative feedback
- Collaboration between institutions to generate question banks
- Managing large groups of students (decreased grading time)
- Availability of student profiles
- Easier access for disabled students.

To this list the advantage of providing for a variety of student assessment preferences should be added. In the study by Engelbrecht and Harding (2003a) the issue of providing for different student assessment preferences, is addressed.

A further important advantage of web-delivered assessment is that it is asynchronous – it need not be restricted to specific locations or times.

An obstacle in using web-based assessment is the time-consuming development of suitable questions and the expertise required. Further disadvantages of web-based assessment identified by the above-mentioned working group (Booth et al, 2001) include set-up costs, the danger of being forced to use a particular type of question as well as the lack of security in assessment situations.

Solutions suggested for the security problem could include accredited assessment centres, the use of passwords, invigilation at summative tests, frequent testing of students (reducing the importance of individual tests) and also a close monitoring of student records to detect wild fluctuations. A new development is the availability of biometric authentication software that is capable of identifying students from the rhythm of their keystrokes. In future, web cameras and voice recognition software could be incorporated into systems to supervise students during tests (Booth et al, 2001). However, currently the authentication of students taking tests at remote locations is still a problem.

VLE systems have an incorporated quiz feature that is used for online assessment. Questions can be asked in a variety of formats, including multiple-choice, matching, short answer and calculated questions where the data is generated. Answers can also be given in paragraph style but then have to be graded individually by the instructor.

One of the big problems in assessing mathematics online is firstly the typing of mathematical symbols by students and secondly the interpretation of mathematical symbolism in student answers. Recent developments (Sangwin, 2003) make the use of a computer algebra system (such as Maple or Mathematica) possible, to be used in interpreting the inputs by students in tests. They have succeeded in incorporating these procedures into the quiz features of VLE systems like WebCT or Blackboard. Once this feature is easily usable, online assessment in mathematics will move into a new era. It will become possible to use open-ended questions such as the following in an online assessment environment – a whole new dimension (Sangwin, 2003).

- Find a quadratic with roots at $x = 1$ and $x = 3$.

- Give examples of differentiable functions, each with a turning point at $x = 1$.
- Find a cubic polynomial $p(x)$ with the following properties:
  (i) $p(0) = 0$, (ii) $p(1)=1$, (iii) p is a bijection of the real line to itself.
- Find a singular 5 x 5 matrix with no repeated entries.

Assessment does not have to consist only of tests, assignments, or computer quizzes. Waldock et al (2001) have introduced the idea of an online *logbook*. Students have to write a few sentences on a weekly basis about each module they are currently registered for, indicating what went well, what did not go well and what plans and steps they intend taking to deal with problems that may have arisen. The objective is to develop students' planning and reflective skills and it has the further advantage for students that they are encouraged to face problems and commit strategies for solving these. To ensure that students participate in these activities, they get marks for the regularity and quality of the logbook entries.

Another way of assessing progress is for students to develop a web-based *portfolio* of their work. During their progress on their degree, students accumulate an online collection of their work. This can include an ongoing resumé and separate pages for each module (Waldock et al, 2001).

5. Benefits of and problems with online mathematics courses

All progress has a price and nothing comes free. Johnson (2003) quotes an appropriate piece of narration in the movie picture *Inherit the Wind*

> All right, you have a telephone; but you lose privacy… Madam, you may vote, but at a price; …you lose your right to retreat behind your petticoat. Mister, you may conquer the air; but… the clouds will smell of gasoline.

In online teaching, exactly the same is true and critics claim that the price is too high. What do we gain and what do we lose? Many of the benefits and problems experienced with online mathematics courses have been discussed in earlier sections. In this section we try to summarise some of the possible advantages and disadvantages.

5.1 Benefits

Range of resources

The wide range of available educational resources is considered to be one of the major benefits of online teaching (Macdonald et al, 2001; Daugherty and Funke, 1998). Students are able to access the most current and global resources, often not even yet available in textbooks or other media. In textbooks information is presented in a linear fashion. The hypertext feature of the internet gives ongoing access to related ideas in other sources. (Tan and Hung, 2002). Students can store and retrieve information effectively by reviewing multiple representations, including readable, printable, searchable documents (Swan et al, 2000, Hopper, 2001). The vast source of interactive and illustrative material available in the mathematics field, such as applets and assessment tools can be used to great advantage for enriching courses. The wide range of resources also offers excellent exploration opportunities for students, especially for stronger students who are interested in mathematics beyond the course work.

Convenience, flexibility and accessibility

Online learning has demonstrated an advantage over the traditional learning environment in the asynchronous sense of learning anytime and anywhere. Continuous accessibility of mathematics courses becomes a reality with the internet being available 24 hours a day, 7 days a week and any place (Tan and Hung, 2002). This means students have access to the online course material independent of time and place (Berge, 1997; Harasim et al, 1997; Matthews, 1999; Swan et al, 2000; Simonson et al, 2000). Prospective students can browse through the course material (in cases where there is no security on the website) (Hopper, 2001).

Continuous accessibility also means that it is available for students who do not necessarily live close to a university, regardless of race, sex, disability or appearance (Suanpang et al, 2003, Matthews, 1999; Swan et al, 2000). Furthermore, Porter (1997) mentions that whereas the best ratio of students to teachers is believed to be no more that 10-15 students per teacher in a face-to-face teaching mode, internet teaching has the potential to liberally increase this number.

Dynamic learning environment

Online teaching supports an active and dynamic learning environment (Macdonald et al, 2001). Course information can be quickly distributed to all students since unlike textbooks, the internet can be dynamically updated (Tan and Hung, 2002) and a confidential student administration (marks) can be conducted online (Hopper, 2001). There is the added advantage of posting handouts and class slides (in face-to-face teaching) on a notice board website that can be accessed by students whenever they choose. In mathematics it is not uncommon for some discussion to develop in class around a certain problem and often there is not enough time to deal with it in class. An accompanying website offers ideal opportunity for the problem exposition to be fully made available for students.

Communication opportunities

By using electronic communication tools, collaborative learning can be done beyond physical and regional boundaries. Interaction between student and content, student and teacher and student and student is unrestricted, the teacher can post a single response to a general content question on an online bulletin board and students can post questions and submit assignments online (Swan et al, 2000). Using electronic discussion forums, e-mail communication or chat rooms, topical discussions can take place (Hopper, 2001). In mathematics the possibility of using a tablet (an electronic writing pad) for quick and effective problem exposition offers an enhancement.

Both synchronous and asynchronous modes of communication have advantages and disadvantages as noted for example by Berge (1999), and Branon and Essex (2001). Synchronous communication has the advantage of participants holding to virtual office hours, it benefits team-decision making, facilitates brainstorming and assists community building. Asynchronous communication has the advantage of encouraging in-depth, more thoughtful discussion, of communicating with diverse students, of allowing students ample time to respond to a topic, of archiving discussions and of students controlling their own pace of learning.

Individual and independent learning

The asynchronous nature of online courses requires of students to make their own decisions on when and where to do what, to reflect upon the materials and their responses and permit students to work at their own pace (Suanpang et al, 2003) whereas in traditional mode the teacher decides on the pace. Students that have been exposed to online mathematics courses, tend to be more academically mature than their counterparts in traditional courses (Engelbrecht and Harding, 2002b).

Natural for today's students

Today's student is used to the internet, it forms an integral part of their lives. Whilst using the internet for teaching and learning could still be somewhat novel to the teacher generation, students find this notion far more natural. In fact, being exposed to courses with no web component may feel somewhat artificial and outdated for students.

5.2 Problems

Staff reluctance

Hopper (2001) relates an appropriate and amusing story about a recent international conference on teaching and learning consisting of a majority of technology-related presentations. Perhaps the most penetrating insight on teaching was delivered in a casual remark by a bartender at the cocktail buffet on the first evening of the meeting. She dryly observed that the party went way over time because the teachers insisted on lining up at the buffet and served themselves one by one. It took forever.

> This seemingly trivial incident hints of the massive weight of anchors of our aggregated paradigms as educators. … It is the way we do business, familiar and benign, and to overcome our collective inertia and see the potential beyond requires great effort (Hopper, 2001).

Some staff members are ardent defenders of the traditional way of teaching and strongly resist implementation of new technologies. Objections include a lack of professional development opportunities for staff to become familiar with the new technologies. Questions are (rightly) raised about the pedagogical merit of the new teaching environment (Duderstadt, 1999). Some educators are strongly of the opinion that online learning isolates the learner.

Lack of face-to-face contact

There is a definite concern about the loss of interpersonal relations in the classroom when teaching via the internet. Czerniewicz (2001), a teacher herself, had a personal experience of being a student in an online course run by a British university. She experienced a "sense of isolation".

> I never did feel that I got to know anyone on the course or that we learned together. (This is not the same as saying that I did not learn from them, but that the learning that took place was not the consequence of collaboration or interaction)…. Indeed, one participant commented, "when you have a small group of what are essentially strangers trying to work together without any common sense of association it might provide with a very negative experience."

She severely experienced the lack of an audience and personal connection and experienced the need to create an "online presence" – another version of herself, her "online self". This required some persistence and behaving in a way different from in face-to-face situations. While normally a good listener, listening is considered lurking in this environment – listening has to be verbal and typed to make other people aware of your presence.

Ng (2001) did a study in which e-mail was used to foster collaboration. Students experienced the absence of a physical presence as a problem. As one of the students said,

> I don't know who has read my work or I have no idea of what people look like.

And another:

> We seldom see each other. When we see each other, we don't have the chance to dine or talk to each other. There is a gap here.

Staff are concerned about the fact that online learning is neither personal nor interactive and is consequently claimed to be inferior to traditional teaching strategies. This is especially true for mathematics that comes with a century long tradition of verbal teaching.

Communication

Disadvantages of asynchronous communication include the lack of immediate feedback, students not checking in often enough, the length of time necessary for discussion to mature and the sense of social disconnection experienced by students.

Synchronous communication negates the "my place and my pace" paradigm, it is problematic to get students online at the same time, there is lack of reflection time for students and moderating large-scale conversations is difficult.

Inexperience and technical problems

The most commonly cited obstacle is the lack of technical expertise required to design, develop and deliver a web-based course (Macdonald et al, 2001). The lack of technical support, of appropriate software and adequate equipment and of administrative support is also considered to be a problem (Daugherty and Funke, 1998). This is true in all fields of study but especially in mathematics with the added concern of symbol presentation.

Immaturity of students

There is no question that in a learner centred environment like an online course, less spoon feeding takes place and more demands are made on the student to make his/her own decisions on when and what inputs to make to ensure progress. The student is assumed to be able to cope with self-directed learning and to be motivated to learn about the topic in question. Concern has been expressed about these assumptions (Tan and Hung, 2002).

Cost

Initially it may seem that the operation of a virtual course is inexpensive, you need a server and a website. However, developing quality web-based courses can become very expensive. Hopper (2001) estimates that an hour of meticulously designed commercial calibre web-based instruction may require 400 to 500 hours of skilled labour. However, with the availability of VLEs, this is probably somewhat of an overestimate.

6. Research questions

The small amount of research done on web-assisted or web-based mathematics courses is mainly in the form of case studies. For that matter, even on using computers in general in teaching undergraduate mathematics (not in a web-based environment) little research of a quantitative nature has been published.

The lack of research in this field will probably not prevent the use of the internet in teaching mathematics and the research may rather reflect on practice than drive the practice. In fact, already huge investments (time and money) have been made in developing online academic programmes before the question of efficacy has been explored.

Because an online pedagogy does not exist at this stage, we need an understanding of how people learn online and online learning styles. We need to identify the pedagogical elements that might be preserved in an online context (Johnson, 2003).

We list some research questions that are currently topical.

Student preferences

With a first generation of web-based courses it is expected that teachers would be interested in knowing regarding the student preferences about web-based courses versus traditional courses (Stokes, 2001; Engelbrecht and Harding, 2001a). Generally it is agreed that although students are reluctant to newly enter a web-based course, they tend to view it positively after having been exposed to the environment. Stokes (2001), who investigated the impact of students' temperaments on their preference, reports

> The majority of participants in this study expressed satisfaction regardless of age, grade, point average, university classification, major, and experience that incorporate web-based lessons.

Although some work has been done in this regard, this is definitely not a well-researched issue.

Students as independent learners

A major objective in teaching post-secondary students is to move students from dependency on their teacher to self-reliance and to develop their ability to learn independently within their chosen field (Berge, 1999). Leh (2001) did a study in which she employed a variety of teaching strategies that would encourage students to become independent learners. Engelbrecht and Harding (2002b) are doing a study using a qualitative as well as a quantitative measuring instrument to try to ascertain the impact that web-based teaching has on the development of academic maturity in students. Questions include the importance of student self-monitoring – how and why do students make time management decisions, for example, dividing their time among reflection, estimating and checking, calculating (Bookman and Malone, 2002).

Role of the instructor

The role of the instructor in this teaching mode is still unclear. Bookman and Malone (2002) list a number of issues in this regard such as when and how the instructor should intervene, support, and guide students in their work and how the instructor can facilitate productive dialogue among students,

Internet implementation

In part I of this paper (Engelbrecht and Harding, 2004) a number of different ways have been described to use the internet in teaching mathematics. Pea (1987) distinguishes between using the computer as an amplifier or as an organiser. The internet can be used to amplify the speed or quality of existing activities, as an example by performing computations that are difficult by hand, or symbolic manipulations that are often cumbersome using a CAS system. Using the course website as an organiser changes the nature of the activities. We do not know yet to what extent we should include the internet in our normal teaching.

Communication and collaboration

It is clear from what we discussed in section 3 that the internet is not a social learning environment. The success of electronic communication in a mathematics learning environment has to be addressed. We need an articulation of the types and values of interaction: learner/content, learner/instructor, and learner/learner. We need an examination of what elements of human interaction might be lost and how that loss might be mitigated (Johnson, 2003). Questions such as if and how a virtual community can successfully function within the current web-based technologies without face-to-face contact and how to develop facilitator and moderator techniques to compensate for some of the problems with online communication, should be addressed (Johnson, 2001). Another issue is whether interdependence and shared responsibility, as well as other aspects of cooperative learning, can be built into online courses (Bookman and Malone, 2002).

Visual internet tools

Using the computer to enhance visualisation is a popular topic for research in mathematics education (Tall, 1994). Although pictures do not always represent rigorous mathematics, for a large portion of mathematics undergraduates the use of visualisation represents a bridge towards proper understanding (Crowe and Zand, 2000). Many courses use Java applets but not enough has been done to establish how effective these visual internet tools are for students learning mathematics. Issues to be investigated include how, when, and why students choose among their tools, such as paper and pencil, calculator, computer algebra system or visual demonstration and also how well the tools enable students to avoid time-consuming calculations, and how the saved time is spent. (Bookman and Malone, 2002).

Assessment

Features of online assessment has been investigated by a number of people (e.g. Booth et al, 2001, Engelbrecht and Harding, 2003a, 2003b). The possibilities in online tests, however, are still limited. Question formats that make provision for constructive mathematical responses should be developed and the typing of mathematical symbolism and the evaluation of mathematical responses from students have to be evaluated (Sangwin, 2003). Security is also still a problem and measures have to be developed to make sure that students do their own tests – even without human invigilation.

Is it for everybody?

Kearsley (2002) expresses the opinion that online learning is not for all students, not for all teachers, not for all content, not for all administrators and not for all cultures. Students without the necessary self-discipline and study skills find the teaching medium frustrating. Excellent classroom teachers do not necessarily make good online instructors and furthermore institutions should not insist that all instructors teach online. Allen et al (1998) are of the opinion that only students that are strongly motivated self-starters, intellectually mature, home-schooled, or the handicapped can successfully complete the on-line mathematics course.

A study by Ory et al, (1997) reports that a few small-in-magnitude gender differences, including the fact that female students use computers more often for conferencing with the instructor and other students but less often for exploring resources on the web, exist. They also find using computers to be slightly more difficult, they are less likely to use personal computers in their apartment or residence hall room and reported greater gains in their familiarity with computers after taking an online course. Apart from these small issues, there were no significant gender differences in online learning or attitudes toward online learning.

Becker and Dwyer (1998) found that more visually inclined learners tend to prefer the use of technology while more verbal learners preferred a face-to-face learning environment.

In mathematics there is, on top of the issues raised above, also still uncertainty whether all mathematics can be successfully taught via the web.

7. Future Trends

Teaching practices are changing as d-learning (distance learning) is being replaced by e-learning (electronic learning), which in turn is making way for m-learning (mobile learning). The three modes are not exclusive and in all likelihood will blend even more in future. Interestingly, these three stages of development correspond to the influence on society of the Industrial Revolution of the 18th to 19th centuries that made transport and postage possible, the Electronics Revolution of the 20^{th} century that brought us computers and the Wireless Revolution around the start of the 21^{st} century that brought us cellular phones.

Distinction between contact and distant education is rapidly disappearing and practices are becoming integrated. Most courses already have an online component and we predict that this will soon be the norm. More and more of the scheduled activities could be happening online with perhaps few scheduled face-to-face sessions. Distance courses could all be presented online with multimedia attributes that create a virtual reality not much different from the traditional contact courses, teaching face-to-face at a distance.

In a paper on developing an m-learning model for Africa, Brown (2003) talks about a new literacy emerging, that of information navigation. You will become your own personal librarian and navigate your information personally. Students already see the internet as their primary source of information. The move towards wireless technology will gain more momentum and m-learning is the buzz word for the foreseeable future. The educational market for m-learning is two-fold according to Brown (2003). On the one hand there are the learners that are continually on the go and cannot afford to be in any one place for too long. On the other hand there are the learners that are without the infrastructure of computer laboratories and cannot afford personal computers. The first group will benefit from the mobility of wireless technology and the second group will benefit from the affordability. The challenge is to develop didactic environments for mobile phones and mobile computers and other yet unknown mobile devices as this technology blossoms.

Dye (2000), on discussing the future of online mathematics, says that his vision for fifteen years on would be one seamless online medium for doing mathematics interactively. This medium would involve at its centre some future development or evolution of the browser, whereby many and varied types of mathematical activity, ranging from geometric and graphical to algebraic and statistical, are all handled by the same interface. No longer will the student have to learn first how to operate the technical aspects of many different pieces of software; instead they will be able to choose their own route through a mathematical environment, capable at any stage of doing mathematics immediately. This is not a revolution about technology at all but about accessibility of content and resources. This environment will be a changing and developing medium produced globally by teams of educators and programmers to fit to agreed rules or principles of operation.

As for undergraduate mathematics, it is reasonably safe to predict development of browsers that are totally compatible with symbol presentation, virtual learning environments with a computer algebra system integrated into it, whiteboard features as standard practice with voice communication. Virtual libraries will expand and present course material, illustrative and exploratory material that can be used to easily design an interactive course from your desktop.

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